FEDERAL PUBLIC SERVICE COMMISSION

**COMPETITIVE EXAMINATION FOR
RECRUITMENT TO POSTS IN BS-17
UNDER THE FEDERAL GOVERNMENT, 2013**

APPLIED MATHEMATICS, PAPER-I

TIME ALLOWED: THREE HOURS	MAXIMUM MARKS: 100

NOTE: **(i)** Candidate must write **Q.No.** in the **Answer Book** in accordance with **Q.No.** in the **Q.Paper**.
(ii) Attempt **FIVE** questions in all by selecting **THREE** questions from **SECTION-A** and **TWO** questions from **SECTION-B**. All questions carry **EQUAL** marks.
(iii) **Use of Calculator is allowed.**
(iv) Extra attempt of any question or any part of the attempted question will not be considered.

SECTION-A

Q.1. **(a)** Find a function φ such that $\nabla\varphi = \overleftrightarrow{f}$ **(10)**

$$\overleftrightarrow{f} = x\hat{i} + 2y\hat{j} + 2\hat{k}$$

(b) Prove that **(10)**

$$\nabla\varphi^n = n\varphi^{n-1}\nabla\varphi$$

Q.2. **(a)** Show that for any vectors $\vec{a}$ and $\vec{b}$ **(10)**

$$\left|\vec{a}+\vec{b}\right|^2 + \left|\vec{a}-\vec{b}\right|^2 = 2\left(\left|\vec{a}\right|^2 + \left|\vec{b}\right|^2\right)$$

(b) Prove that **(10)**

$$\left(\vec{a}\times\vec{b}\right)\bullet\left(\vec{b}\times\vec{c}\right)\times\left(\vec{c}\times\vec{a}\right) = \left(\vec{a}\bullet\vec{b}\times\vec{c}\right)^2$$

Q.3. **(a)** The greatest result that two forces can have is of magnitude P and the least is of magnitude Q. Show That when they act an angle α their resultant is of magnitude **(10)**

$$\sqrt{P^2\cos^2\alpha/2 + Q^2\sin^2\alpha/2}$$

(b) A uniform rod of length $2a$ rests in equilibrium against a smooth vertical wall and upon a smooth peg at a distance b from the wall. Show that in the position of equilibrium the rod is inclined to the wall at an angle $\sin^{-1}\left(\dfrac{b}{a}\right)^{\frac{1}{3}}$ **(10)**

Q.4. **(a)** Three forces P, Q and R act along the BC, CA and AB respectively of triangle ABC. Prove that if $P\cos A + Q\cos B + R\cos C = 0$, then the line of action of the resultant passes through the circum center of the triangle. **(10)**

(b) A sphere of weight W and radius a is suspended by a string of length l from a point P and a weight w is also suspended from P by a string sufficiently long for the weight to hang below the sphere. Show that the inclination of the first string to the vertical is **(10)**

$$\sin^{-1}\left(\frac{wa}{(W+w)(a+l)}\right)$$

APPLIED MATHS, PAPER-I

Q.5.

(a) Find the volume $\iint_R \left(x^3 + 4y \right) \, dA$ where R is the region bounded by the parabola $y = x^2$ and the line $y = 2x$. **(10)**

(b) Evaluate the following line integral **(10)**

$$\int_c x^2 \, dy$$

bonded by the triangle having the vertices (-1,0) to (2,0), and (1,1)

SECTION-B

Q.6.

(a) The position of a particle moving along an ellipse is given by $\overleftrightarrow{r} = a \cos t \, \hat{i} + b \sin t \, \hat{j}$. If $a > b$, find the position of the particle where its velocity has maximum or minimum magnitude. **(10)**

(b) Prove that the speed at any point of a central orbit is given by: **(10)**

$$vp = h,$$

When h is the areal speed and p is the perpendicular distance from the centre of force, of the tangent at the point, Find the expression for v when a particle subject to the inverse square law of force describes an ellipse, a parabolic and hyperbolic orbit.

Q.7. (a) A particle is moving with the uniform speed v along the curve **(10)**

$$x^2 y = a \left(x^2 + \frac{a^2}{\sqrt{5}} \right)$$

Show that its acceleration has the maximum value at $\frac{10v^2}{9a}$

(b) An aeroplane is flying with uniform speed v_0 in an arc of a vertical circle of radius a, whose centre is a height h vertically above a point O of the ground. If a bomb is dropped from the aeroplane when at a height Y and strikes the ground at O, show that Y satisfies the equations **(10)**

$$KY^2 + Y \left(a^2 - 2hK \right) + K \left(h^2 - a^2 \right) = 0,$$

where $K = h + \dfrac{ga^2}{2v^2_0}$

Q.8. (a) Find the tangential and normal components of the acceleration of a particle describing the ellipse **(10)**

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

With uniform speed v when the particle is a $a > b$

(b) Find the velocity acquired by a block of wood of mass M lb., which is free to recoil when it is struck by a bullet of mass m lb. moving with velocity v in a direction passing through the centre of gravity. If the bullet is embedded a ft., show that the resistance of the wood to the bullet, supposed uniform, is $\dfrac{Mm^2}{2(M+m)ga}$ lb.wt. and that the time of penetration is $\dfrac{2a}{v}$ sec., during which time the block will move $\dfrac{ma}{m+M}$ ft. **(10)**

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Roll Number

APPLIED MATHEMATICS, PAPER-II

TIME ALLOWED: THREE HOURS **MAXIMUM MARKS: 100**

NOTE: **(i)** Candidate must write **Q. No.** in the **Answer Book** in accordance with **Q. No.** in the **Q. Paper.**
(ii) Attempt **FIVE** questions in all by selecting **TWO** questions from **SECTION-A** and **ONE** question from **SECTION-B** and **TWO** questions from **SECTION-C ALL** questions carry **EQUAL** marks**.**
(iii) Extra attempt of any question or any part of the attempted question will not be considered.
(iv) **Use of Calculator is allowed.**

SECTION-A

Q.No.1. Solve the following equations:

(a) $\dfrac{d^3 y}{dx^3} + \dfrac{dy}{dx} = Sec^2 x$ **(10)**

(b) $\dfrac{2dy}{dx} - \dfrac{x}{y} + x^3 Cos\ y = 0$ **(10)**

Q.No.2. **(a)** Find the power series solution of the differential equation **(10)**
$(1 - x^2)y^{//} - 2xy^{/} + 2y = 0$, about the point $x = 0$.

(b) Solve $Z(x+y)\dfrac{\partial Z}{\partial x} + Z(x-y)\dfrac{\partial Z}{\partial y} = (x^2 + y^2)$. **(10)**

Q.No.3. **(a)** Classify the following equations: **(5)**

(i) $\dfrac{\partial^2 Z}{\partial x^2} + x^2 \dfrac{\partial^2 Z}{\partial y^2} - \dfrac{1}{x}\dfrac{\partial Z}{\partial x} = 0$

(ii) $x^2 \dfrac{\partial^2 Z}{\partial x^2} + 2xy\dfrac{\partial^2 Z}{\partial x \partial y} + y^2 \dfrac{\partial^2 Z}{\partial y^2} = 4x^2$

(b) Solve: $\dfrac{\partial u}{\partial t} = \dfrac{\partial^2 u}{\partial x^2}, -1 < x < 1, t > 0$ **(15)**

$u(-1, t) = u(1, t);\ \dfrac{\partial u}{\partial x}(-1, t) = \dfrac{\partial u}{\partial x}(1, t)$ for $t > 0$

$u(x, o) = x+1,\ -1 < x < 1$.

SECTION-B

Q.No.4. **(a)** Highlight the difference between a vector and a tensor. What happens if we permute the subscripts of a tensor? **(5)**

(b) Transform $g^{ab} = \begin{pmatrix} 1 & 0 \\ 0 & 1/r^2 \end{pmatrix}$ into Cartesian coordinates. **(15)**

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APPLIED MATHEMATICS, PAPER-II

Q.No.5. **(a)** Workout the Christoffel symbols for the metric tensor $g_{ab} = \begin{pmatrix} a^2 & 0 \\ 0 & a^2\sin^2\theta \end{pmatrix}$ **(10)**

(b) Workout the two dimensional metric tensor for the coordinates p and q given by **(10)**

$$p = (xy)^{\frac{1}{3}}, \quad q = \left(\frac{x^2}{y}\right)^{\frac{1}{3}}$$

SECTION-C

Q.No.6. **(a)** Solve the following system of equations by Jacobi iteration method: **(10)**
$10x + y - 2z = 7.74$
$x + 12y + 3z = 39.66$
$3x + 4y + 15z = 54.8$

(b) Solve $Sin\,x = 1 + x^3$ Using Newton-Raphson method. **(10)**

Q.No.7. **(a)** Find the root of $xe^x = 3$ by regular falsi method correct to three decimal places. **(10)**

(b) Evaluate $\int_0^{10} \dfrac{dx}{1+x^2}$ using **(5+5)** **(10)**

    (i) Trapezoidal rule and
    (ii) Simpson's rule.

Q.No.8. **(a)** Find the real root of the equation $Cos\,x = 3x - 1$ correct to seven decimal places by the iterative method. **(10)**

(b) Use Lagrange's interpolation formula to find the value of y when $x = 10$, if the values of x and y are given below: **(10)**

X	5	6	9	11
Y	12	13	14	16

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