FEDERAL PUBLIC SERVICE COMMISSION

Roll Number

COMPETITIVE EXAMINATION FOR RECRUITMENT TO POSTS IN BS-17 UNDER THE FEDERAL GOVERNMENT, 2011

APPLIED MATHEMATICS, PAPER-I

TIME ALLOWED: THREE HOURS	MAXIMUM MARKS: 100

NOTE: **(i)** Attempt **FIVE** questions in all by selecting **THREE** questions from **SECTION – A** and **TWO** questions from **SECTION – B.** All questions carry equal marks.
(ii) **Use of Scientific Calculator is allowed.**
(iii) **Extra attempt of any question or any part of the attempted question will not be considered.**

SECTION - A

Q.1. (a) Find the divergence and curl $\vec{f}$ If $\vec{f} = 2xyz\hat{i} + (x^2z + 2y)\hat{j} + (x^2y + 3z^2)\hat{k}$ **(10)**

(b) Also find a function φ such that $\nabla\varphi = \vec{f}$ **(10)**

Q.2. (a) Find the volume $\iint_R xy\, dA$ where R is the region bounded by the line y = x – 1 and the parabola $y^2 = 2x + 6$. **(10)**

(b) Evaluate the following line intergral: **(10)**

$\int_C y^2dx + xdy$ *where* $c = c_2$ is the line segment joining the points (-5, -3) to (0, 2), and c = c_2 is the arc of the parabola x = 4 - y^2.

Q.3. (a) Three forces P, Q and R act at a point parallel to the sides of a triangle ABC taken in the same order. Show that the magnitude of the resultant is **(10)**

$\sqrt{p^2 + Q^2 + R^2 - 2QR\cos A - 2RP\cos B - 2PQ\cos C}$

(b) A hemispherical shell rests on a rough inclined plane whose angle of friction is λ. Show that the inclination of the plane base to the horizontal cannot be greater than arcsin(**2 sin** λ) **(10)**

Q.4. (a) A uniform square lamina of side $2a$ rests in a vertical plane with two of its sides in contact with two smooth pegs distant b apart and in the same horizontal line. Show that if $\frac{\theta}{\sqrt{2}} < b < a$, a non symmetric position of equilibrium is possible in which $b(\sin\theta + \cos\theta) = a$ **(10)**

(b) Find the centre of mass of a semi circular lamina of radius a whose density varies as the square of the distance from the centre. **(10)**

Q.5. (a) Evaluate the integral $\int_0^1 \int_{x^2}^{x} (x^2 + y^2)dydx$ **(10)**

also show that the order of integration is immaterial.

(b) Find the directional derivative of the function at the point P along z – axis **(10)**
$f(x, y) = 4xz^3 - 3x^2y^2z, P = (2,-1,2)$

SECTION – B

Q.6. (a) A particle is moving along the parabola $x^2 = 4ay$ with constant speed v. Determine the tangential and the normal components of its acceleration when it reaches the point whose abscissa is $\sqrt{5}a$ **(10)**

(b) Find the distance travelled and the velocity attained by a particle moving in a straight line at any time t, if it starts from rest at t =0 and is subject to an acceleration $t^2 + \sin t + e^t$ **(10)**

Q.7. (a) A particle moves in the xy – plane under the influence of a force field which is parallel to the axis of y and varies as the distance from x – axis. Show that, if the force is repulsive, the path of the particle supposed not straight and then **(10)**

$$y = a \cosh nx + a \sinh nx$$

where a and b are constants.

(b) Discuss the motion of a particle moving in a straight line with an acceleration x^3, where x is the distance of the particle from a fixed point O on the line, if it starts at t = 0 from a point x = c with the velocity $c^2/\sqrt{2}$. **(10)**

Q.8. (a) A battleship is steaming ahead with speed V and a gun is mounted on the battleship so as to point straight backwards and is set at angle of elevation α. If v_0 is the speed of projection (relative to the gun) show that the range is $\frac{2v_0}{g}\sin\alpha(v_0\cos\alpha - V)$ **(10)**

(b) Show that the law of force towards the pole of a particle describing the survey $r^n = a^n \cos n\theta$ is given by $f = \frac{(n+1)h^2a^{2n}}{r^{2n+3}}$ where h is a constant. **(10)**

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APPLIED MATHEMATICS, PAPER-II

TIME ALLOWED: THREE HOURS	**MAXIMUM MARKS: 100**

NOTE: **(i)** Attempt **FIVE** questions in all by selecting **THREE** questions from **SECTION – A** and **TWO** questions from **SECTION – B.** All questions carry equal marks.
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SECTION - A

Q.1. (a) Solve by method of variation of parameter **(10)**

$$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = xe^x 1nx$$

(b) Solve first order non-linear differential equation **(10)**

$$x\frac{dy}{dx} + y = y^2 1nx$$

Q.2. (a) Solve **(10)**

$$c^2 u_{xx} = u_{tt}\,.$$

$$u(0,t) = 0$$

$$u(l,t) = 0$$

$$u(x,0) = \lambda\, Sin\left(\frac{\pi}{l}x\right)$$

$$u_t(x,0) = 0$$

(b) Solve $x^2\frac{\partial z}{\partial x} + y^2\frac{\partial z}{\partial y} = (x+y)z$ **(10)**

Q.3. (a) Work out the two dimensional metric tensor for the coordinates p and q given by **(10)**

$$p = (xy)^{\frac{1}{3}},\; q = \left(x^2 / y\right)^{\frac{1}{3}}$$

(b) Prove that $\Gamma^d_{ab} = \frac{1}{2}g^{dc}\left(g_{ac,b} + g_{bc,a} - g_{ab,c}\right)$ **(10)**

APPLIED MATHEMATICS, PAPER-II

Q.4. (a) Work out the Christoffel symbols for the following metric tensor **(10)**

$$g_{ab} = \begin{pmatrix} 1 & 0 \\ 0 & r^2 \end{pmatrix}$$

(b) Work out the covariant derivative of the tensor with components **(10)**

$$\begin{pmatrix} r\cos\theta & ar\sin\varphi & ar \\ \sin\theta\sin\varphi & a\sin\theta\cos\varphi & a \\ \cos\varphi & a\sin\varphi & 0 \end{pmatrix}$$

Q.5. (a) Find recurrence relations and power series solution of $(x-3)y' + 2y = 0$ **(10)**

(b) Solve the Cauchy Euler's equation $x^4 y''' + 2x^3 y'' - x^2 y' + xy = 1$ **(10)**

(10)

SECTION – B

Q.6. (a) Find the positive solution of the following equation by Newton Raphson method **(10)**

$$2 \sin x = x$$

(b) Solve the following system by Jacobi method: **(10)**

$$10x_1 - 8x_2 = -6$$

$$-8x_1 + 10x_2 - x_3 = 9$$

$$-x_2 + 10x_3 = 28$$

Q.7. (a) Evaluate the following by using the trapezoidal rule. **(10)**

$$\int_0^1 (x+1)dx$$

(b) Evaluate the following integral by using Simpson's rule **(10)**

$$\int_0^4 e^x \, dx$$

Q.8. (a) Solve the following equation by regular falsi method: **(10)**

$$2x^3 + x - 2 = 0$$

(b) Calculate the Lagrange interpolating polynomial using the following table: **(10)**

x	0	1	2
f(x)	1	0	-1

also calculate f (0.5).

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