

APPLIED MATH, PAPER-I

**FEDERAL PUBLIC SERVICE COMMISSION
COMPETITIVE EXAMINATION FOR
RECRUITMENT TO POSTS IN BPS-17 UNDER
THE FEDERAL GOVERNMENT, 2009**

APPLIED MATH, PAPER-I

S.No.	
R.No.	

TIME ALLOWED: 3 HOURS**MAXIMUM MARKS:100****NOTE:**

- (i) Attempt **FIVE** question in all by selecting at least **TWO** questions from **SECTION – A** and **THREE** question from **SECTION – B**. All questions carry **EQUAL** marks.
(ii) **Use of Scientific Calculator is allowed.**

SECTION – A

Q.1. (a) Show that in orthogonal coordinates: **(5+5)**

$$(i) \quad \nabla \times (A_1 e_1) = \frac{e_2}{h_1 h_3} \frac{\partial}{\partial u_3} (A_1 h_1) - \frac{e_3}{h_1 h_2} \frac{\partial}{\partial u_2} (A_1 h_1),$$

$$(ii) \quad \nabla \cdot (A_1 e_1) = \frac{1}{h_1 h_2 h_3} \frac{\partial}{\partial u_1} (A_1 h_2 h_3).$$

(b) Write Laplace's equation in parabolic cylindrical coordinates. **(10)**

Q.2. (a) Evaluate $\iint_S \vec{A} \cdot \vec{n} ds$, where $\vec{A} = z\hat{i} + x\hat{j} - 3y^2\hat{k}$ and s is the surface of cylinder $x^2 + y^2 = 16$ included in the first octant between $z=0$ and $z=5$. **(10)**

(b) Verify Green's theorem in the plane for

$$\oint_C (xy + y^2) dx + x^2 dy$$

where C is the closed curve of the region bounded by $y = x$ and $y = x^2$.

Q.3. (a) Find centre of mass of a right circular solid cone of height h . **(10)**

(b) A light thin rod, 4m long, can turn in a vertical plane about one of its point which is attached to a pivot. If weights of $3kg$ and $4kg$ are suspended from its ends it rests in a horizontal position. Find the position of the pivot and its reaction on the rod. **(10)**

SECTION – B

Q.4. (a) Find the radial and transverse components of the velocity of a particle moving along the curve $ax^2 + by^2 = 1$ at any time t if the polar angle $\theta = ct^2$. **(10)**

(b) A particle is projected vertically upwards. After a time t , another particle is sent up from the same point with the same velocity and meets the first a height h during the downward flight of the first. Find the velocity of projection. **(10)**

Q.5. (a) If a point P moves with a velocity v given by

$$v^2 = n^2 (ax^2 + bx + c),$$
show that P executes a simple harmonic motion. Find also, the centre, the amplitude and the time period of the motion. **(10)**

APPLIED MATH, PAPER-I

- (b) A particle of mass m moves on xy -plane under the force (10)

$$\vec{F} = -\frac{k}{r^4} \vec{r},$$

where r is its distance from the origin O . If it starts on the positive x -axis at a distance “ a ” from O with speed v_0 in a direction making an angle θ with the positive x -direction, prove that at time t ,

$$\frac{ma^2 v_0^2 \sin^2 \theta - k}{mr^3}$$

- Q.6.** (a) Define angular momentum and prove that rate of change of angular momentum of a particle about a point O is equal to the torque (about O) of the force acting on the particle. (10)
- (b) Find the least speed with which a particle must be projected so that it passes through two points P and Q at heights h_1 and h_2 , respectively. (10)

- Q.7.** (a) Discuss the polar form of an orbit and prove that when a particle moves under central force, the areal velocity is constant. (10)
- (b) A particle moves under a central repulsive force μ/r^3 and is projected from an apse at a distance “ a ” with velocity V . Show that the equation to the path is (10)

$$r \cos \theta = a$$

and that the angle θ described in time t is

$$\frac{1}{p} \tan^{-1} \frac{pVt}{a},$$

where
$$p^2 = 1 + \frac{\mu}{a^2 v^2}, \quad \mu = GM.$$

- Q.8.** (a) Define the terms moment of inertia and product of inertia, and find the moment of inertia of uniform solid sphere of mass m and radius “ a ”. (10)
- (b) Let AB and BC be two equal similar rods freely hinged at B and lie in a straight line on the smooth table. The end A is struck by a blow P perpendicular to AB . Show that the resulting velocity of A is $3\frac{1}{2}$ times that of B . (10)

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TIME ALLOWED: 3 HOURS**MAXIMUM MARKS:100****NOTE:**

- (i) Attempt **FIVE** question in all by selecting at least **TWO** questions from **SECTION-A**, **ONE** question from **SECTION-B** and **TWO** questions from **SECTION-C**. All questions carry **EQUAL** marks.
- (ii) **Use of Scientific Calculator is allowed.**

SECTION – A

- Q.1. (a)** Using method of variation of parameters, find the general solution of the differential equation.

$$y'' - 2y' + y = \frac{e^x}{x} . \quad (10)$$

- (b) Find the recurrence formula for the power series solution around $x=0$ for the differential equation

$$y'' + xy = e^{x+1} . \quad (10)$$

- Q.2. (a)** Find the solution of the problem (10)

$$u'' + 6u' + 9u = 0$$

$$u(0) = 2, \quad u'(0) = 0$$

- (b) Find the integral curve of the equation

$$xz \frac{\partial z}{\partial x} + yz \frac{\partial z}{\partial y} = -(x^2 + y^2) . \quad (10)$$

- Q.3. (a)** Using method of separation of variables, solve (10)

$$\frac{\partial^2 u}{\partial t^2} = 900 \frac{\partial^2 u}{\partial x^2} \quad \begin{cases} 0 < x < 2 \\ t > 0 \end{cases} ,$$

subject to the conditions

$$u(0, t) = u(2, t) = 0$$

$$u(x, 0) = 0 \quad \frac{\partial u}{\partial t} \Big|_{t=0} = 30 \sin 4\pi x .$$

- (b) Find the solution of (10)

$$\frac{\partial^2 u}{\partial x^2} - 2 \frac{\partial^2 u}{\partial x \partial y} + \frac{\partial^2 u}{\partial y^2} = 4e^{3y} + \cos x .$$

SECTION – B

- Q.4. (a)** Define alternating symbol ϵ_{ijk} and Kronecker delta δ_{ij} . Also prove that (10)

$$\epsilon_{ijk} \epsilon_{lmk} = \delta_{il} \delta_{jm} - \delta_{im} \delta_{jl} .$$

- (b) Using the tensor notation, prove that

$$\nabla \times (\vec{A} \times \vec{B}) = \vec{A}(\nabla \cdot \vec{B}) - \vec{B}(\nabla \cdot \vec{A}) + (\vec{B} \cdot \nabla) \vec{A} - (\vec{A} \cdot \nabla) \vec{B} \quad (10)$$

APPLIED MATH, PAPER-II

Q.5. (a) Show that the transformation matrix

$$\mathbf{T} = \begin{bmatrix} -\frac{1}{\sqrt{2}} & \frac{1}{2} & -\frac{1}{2} \\ 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{2} & -\frac{1}{2} \end{bmatrix}$$

is orthogonal and right-handed.

(10)

(b) Prove that

(10)

$$l_{ik} l_{jk} = \delta_{ij}$$

where l_{ik} is the cosine of the angle between i th-axis of the system K' and j th-axis of the system K .

SECTION – C

Q.6. (a) Use Newton's method to find the solution accurate to within 10^{-4} for the equation $x^3 - 2x^2 - 5 = 0$, $[1, 4]$.

(10)

(b) Solve the following system of equations, using Gauss-Siedal iteration method

(10)

$$\begin{aligned} 4x_1 - x_2 + x_3 &= 8, \\ 2x_1 + 5x_2 + 2x_3 &= 3, \\ x_1 + 2x_2 + 4x_3 &= 11. \end{aligned}$$

Q.7. (a) Approximate the following integral, using Simpson's $\frac{1}{3}$ rules

(10)

$$\int_0^1 x^2 e^{-x} dx.$$

(b) Approximate the following integral, using Trapezoidal rule

(10)

$$\int_0^{\pi/4} e^{3x} \sin 2x dx.$$

Q.8. (a) The polynomial

(10)

$$f(x) = 230x^4 + 18x^3 + 9x^2 - 221x - 9$$

has one real zero in $[-1, 0]$. Attempt approximate this zero to within 10^{-6} , using the Regula Falsi method.

(b) Using Lagrange interpolation, approximate.

(10)

$$f(1.15), \text{ if } f(1) = 1.684370, f(1.1) = 1.949477, f(1.2) = 2.199796, f(1.3) = 2.439189, \\ f(1.4) = 2.670324$$
